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Analogicity in Computer Science. Some Methodological and Philosophical Issues.

This text, enriched with some new elements by Paula Quinon, is an extract from two articles by Paweł Stacewicz: 1) "On different meanings of analogicity in computer science" (already published in Polish, in the journal "Semina Scientiarium")[1], 2) "Analogicity in Computer Science. A Methodological Analysis" (submitted and currently reviewed in the journal "Studies in Logic, Grammar and Rhetoric").
Therefore it is not a fully original work.

Nevertheless, we decided to submit it for discussion as part of the workshop "Computational Modeling II" (organized in Cracow, on 11.03.2019, at the UPJPII University), because we are now working on a new publication devoted to analog/continuous computations, and all additional critical input, and each additional discussion will be for us very precious.

Thus, we will be grateful for any comments that may contribute both: the improvement of the text no. 2 (which still is in the reviewing process), and the development of our new ideas.

1. Analogicity in computer science

1.1. Two basic (general) meanings of analogicity

Regardless of the (technical) aspect that is considered in contemporary computer science there exist two different (yet not necessarily separate) ways of understanding analogicity.

[1] Cf. Stacewicz P., „O różnych sposobach rozumienia analogowości w informatyce”, *Semina Scientiarum*, nr 16, 2017, ss. 121-137.

The first meaning, we shall call it AN-A, refers to the concept of *analogy*. It acknowledges that analog computations are based on natural analogies and consist in the realisation of natural processes which, in the light of defined natural theory (for example physical or biological), correspond to some mathematical operations.[2] Metaphorically speaking, if we want to perform a mathematical operation with the use of a computational system, we should find in nature its *natural analogon*. It is assument that such an analogon simply exists in nature and provides the high effectiveness of computations. The initial examples of AN-A techniques (that will be developed later) are: the calculation of quotient using the Ohm's law (an illustrative example) or the integration of functions using physical integrators (a realistic example).

The second meaning, we shall call it AN-C, refers to the concept of *continuity*. Its essence is the generalisation (broadening) of digital methods in order to make not only discrete (especially binary) but also continuous data processing possible.[3] On a mathematical level, these data correspond to real numbers from a certain continuum (for example, an interval of a form [0,1]), yet on a physical level – certain continuous measurable variables (for example, voltage or electric potentials).[4]

In a short comment to this distinction, we would like to add that the meaning of AN-A has, on the one hand, a historical character because the techniques, called *analog*, which consisted in the use of specific physical processes to specific computations, were applied mainly until the 1960s. On the other hand, it looks ahead to the future – towards computations of a new type that are more and more often called *natural* (for example, quantum or computations that use DNA[5]). The meaning of AN-C, by contrast, is more related to mathematical theories of data processing (the theoretical aspect of computations) than to their physical realisations. Perhaps, it is solely a theoretical meaning that, in practice, is reduced to discreteness/digitality (wedevelop this subject in section 2.2) due to physical features of data carriers.

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[2] Cf. G. Ifrah, *Historia powszechna cyfr* [Universal history of numbers], vol. 2, Warsaw 2006, p. 655 [English edition: G. Ifrah, *Universal History of Numbers: From Prehistory to the Invention of the Computer*, translated by D. Bellos, E.F. Harding, S. Wood and I. Monk, London 1998].

[3] Sometimes, especially in some informal contexts, *discrete vs continuous* (the mathematical aspect) and *digital vs analog* (the computer science's aspect) distinctions are treated as oppositions. From a formal point of view, however, we cannot talk about opposition, but complement or extension. A continuous domain includes a discrete domain as its subset, thus a discrete domain can be extended to a continuous domain. For example, a closed interval of a form [0,1] includes a set {0,1}, thus this set {0,1} can be extended to the interval [0,1].

[4] Cf. J. Mycka, M. Piekarz, *Przegląd zagadnień obliczalności analogowej* [A review of the issues of analog computability], in *Algorytmy, metody i programy naukowe* [Algorithms, methods and scientific programmes], eds. S. Grzegórski, M. Miłosz, P. Muryjas, Lublin 2004, pp. 125–132.

[5] Cf. L. Kari, G. Rozenberg, "The many facets of natural computing," *Communications of the ACM* 51 (10) (2008): 72–83.

Additionally, it is important to note that analogousness does not exclude continuity. This means that both continuous and discrete signals can be processed as analogons. Therefore, the above-differentiated meanings are not completely opposed.

1.2. Analogicity in relation to analogousness

The essence of AN-A analog techniques can be called *analogousness* – that is the necessity to use some natural analoga/equivalents of the performed mathematical operations for computing purposes. Such computations are of definitely more *empirical* character than digital techniques which refer to extremely simple states/phenomena.[6] Their specificity can be presented by the following points: a) find in nature a distinct process that "calculate something" (and is described by a certain mathematical formula), b) build a computational system that uses such a process, c) initiate computations configuring the system, d) take measurements in the system and interpret the outcome as the results of computations.[7]

It should be underlined that computations arranged in such a way are always justified by a special *physical theory*, which combines performed mathematical operations with phenomena used for their performance. Based on such a theory, it could be acknowledged that a given phenomenon has such and such mathematical description; and on the contrary: that operations constituting such a description can be performed physically within the phenomenon (or more accurately: the results of these operations can be identified with the results of suitable measurements). This can be explained in more details by the following illustrative (not realistic) example.

> **Example 1.** Let's consider the calculation of quotient using the Ohm's law (I=V/R). This law describes the flow of current in an electrical circuit (it must be added that this is an idealised circuit and its description does not take into considerations such factors as, for example, the self-inductance[8]).
>
> An analog computation is performed in the following way: a) adjust the voltage V and the resistance R appropriately, b) initiate the flow of current, c) take a measurement of current intensity I, interpreting the result as the value of the quotient.

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[6] Obviously, in the case of digital computations two opposite states, such as voltage and the lack of voltage, are sufficient.

[7] Cf. G. Ifrah, *Historia powszechna cyfr*, p. 656.

[8] Cf. W. Krajewski, *Prawa nauki. Przegląd zagadnień metodologicznych i filozoficznych* [Laws of science. A review of methodological and philosophical topics], Warsaw 1998, p. 109.

A physical analogon of the computation is the flow of current in a circuit, which is initiated, controlled and observed with some intention, whereas the theory justifying the computation is the theory of current flow in a conductor (the idealising Ohm's law constitutes its element).

It should be provisionally noted (we will come back to this issue at the end of the text) that the validity and accuracy of AN-A_a analog computations must depend on the level of adequacy of the theory which describes the process that is the basis for computations (in this case: the flow of current through a conductor). If the theoretical mathematical formula (I=V/R) characterises the above-mentioned process good enough (it should be remembered that such formulas refer to idealised situations), the computation can be acknowledged as accurate enough.

The most important characteristic of computations represented by the above example – a characteristic which accounts for the name "analog computations" – is *analogousness*. Such a quality is manifested on two levels.

Firstly, as it is clearly presented by the above example, the process which performs a computation is a physical analogon of a certain mathematical operation. On this level, therefore, we encounter correspondence [a formal operation – a physical phenomenon]. (In the example, this occurred between the calculation of quotient and the flow of current.)

Secondly, the *basic* process that is used to perform a computation can be applied to issues related to *similar physical processes*. Such a possibility is provided by the analogy between these two processes: they are analogic because an identical formal model describes them together. To explain this, we should take into consideration an integrating physical system (an integrator) that is based on a physical process A. Due to the fact that this process is formally described by a theory of integration (for example, the Riemann's one), it can be used in computations that concern a broad category of other processes (B, C, D…) described by this or that integral.[9]

Finally, one more issue leads directly to the second meaning of analogicity. *Continuity* is very often perceived as a crucial feature of AN-A analog computations. This is underlined because early (and specialised) analog systems were used, above all, to solve analytical problems (concerning, for example, differential equations) that are defined and described with the use of continuous real numbers and mathematical structures (such as differentiable

[9] See G. M. Fichtenholz, *Rachunek różniczkowy i całkowy* [Differential and Integral Calculus], vol. 2, Warsaw 1997.

functions) "constructed" upon them.[10] Moreover, processes that performed computations inside the discussed systems were characterised mathematically with continuous (analytical) objects.[11] Despite this, as explained earlier, the essence of AN-A analog techniques does not come down to their continuity.

1.3. Analogicity in relation to continuity

The above-mentioned concept of continuity constitutes a basis for determining the second meaning of analogicity, that is AN-C. From the point of view of modern computer science, this meaning should be recognised as dominant, which is manifested by a common tendency for identifying analog computations with *continuous computations*[12], or, in other words, defining analog computations in opposition to digital computations. The essence of the former is sought in the fact that they make it possible to process and generate continuous (not only discrete) data represented in practice by continuous physical quantities. In short: AN-C analogicity is defined within the distinction *discrete–continuous*.[13]

Having in mind both above-analysed meanings, it should be stated that in the methodology of computer science as well as in the general understanding of IT users, a crucial shift in meanings has occurred: from analogicity understood as analogousness towards analogicity understood as continuity. It was produced due to an engineer's practice, in the result of which more and more *universal* analog machines were created and analog devices were gradually supplanted by *digital* machines that were more reliable.[14]

Aiming at widening the scope of application of analog machines, the first process consisted in searching such a *minimal* set of processing components whose different arrangements (connections) would guarantee the performance of the broadest class of functions. Typical "minimal" components were amplifiers and integrators – adjusted to

[10] Cf. G. Ifrah, *Historia powszechna cyfr*, pp. 651–660.
[11] However, it is unknown whether continuity (of, for example, real numbers) is an actual feature of the above-mentioned processes as well as their results (results of measurements). Perhaps, this is only a feature of their mathematical description. Still, it is known that measuring instruments used in practice force discreteness of the obtained results just because they have finite accuracy. We will return to these issues at the end of the text.
[12] See for example J.F. Costa, D. Graça, "Analog computers and recursive functions over the reals," *Journal of Complexity* 5 (2003): 644–664.
[13] It should be noticed that the distinction discrete–continuous (not being an opposition; see footnote 5) has its foundations in mathematics, in which discrete and continuous objects (for example discrete and continuous sets, discrete and continuous random variables, etc.) are distinguished in a rather standard way. The basis for the definition of the former are natural numbers (N), whereas of the latter – real numbers (R). Even at the level of naming the whole fields of mathematics, it is more and more common to differentiate discrete mathematics from continuous quantity mathematics (based on analysis).
[14] Cf. G. Ifrah, *Historia powszechna cyfr*, pp. 651–662.

process continuous signals. Because of the growing popularity of such solutions, analogicity gradually became identified with *continuity* of processed data. The characteristic feature of analogousness (that is the use of natural analoga to computations) receded into the background because universal analog devices performed certain combinations of several basic operations on continuous signals.[15]

The second process – which consists in the far-reaching universalisation of machines, connected to the digitisation of signals and computations – caused, on the basis of counterbalance, that analog techniques being competitive with digital techniques became associated with, above all, *non-digitality* (non-discreteness), that is with continuity.

The contemporary concept of analogicity in the sense of AN-C can be precisely expressed by *mathematical models* of analog-continuous computations.[16] Following [Mycka, Piekarz 2004], they can be divided into: 1) continuous time models of continuous computation – for example, the GPAC model[17] and 2) models of continuous computations performed in discrete steps – for example, the BSS model[18].

Because of both the idea of "full continuity" (that is the lack of any discretisation, also in the time aspect) and the historical antecedence, the GPAC model seems to be the worthiest of consideration.[19] This model describes the way of processing continuous signals (mathematically speaking: continuous functions) with a minimum number of functional operations that at the level of model constitute nodes/vertices of an oriented graph which joints input and output signals. Such a graph presents the order in which an input signal is to be processed (sometimes simultaneously) by the components of the system that correspond to particular nodes. The before mentioned minimum number of operations includes: multiplying

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[15] Cf. J. Mycka, *Obliczenia dyskretne i ciągłe jako realizacje antropomorficznej i fizycznej koncepcji efektywnej obliczalności* [Discrete and continuous computations as realisations of an anthropomorphic and physical concept of effective computability], in *Światy matematyki. Tworzenie czy odkrywanie* [Worlds of mathematics. Creating or discovering], eds. I. Bondecka-Krzykowska, J. Pogonowski, Poznań 2010, pp. 247–260.

[16] Models of this type should be distinguished from models of analog-analogous computations (in the sense of AN_a), which are in fact fragments of some physical theories. These theories connect defined mathematical structures (including: a mathematical operation performed by a given system) with defined physical processes. Due to the dedicated, hence non-universal character of computations of this type, the corresponding models can be called micro-models (they are not general).

[17] See C. Shannon C., "Mathematical Theory of the Differential Analyzer," *J. Math. Phys. MIT* 20 (1941): 337–354.

[18] See L. Blum, M. Shub, S. Smale, "On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines," *Bull. Amer. Math. Soc. (NS)* 21 (1989): 1–46.

[19] Its methodological significance is supported by the fact that it has some valuable contemporary expansions, for example, the EAC model based on recursive real-valued functions. On the EAC model, see L. Rubel, "The extended analog computer," *Advances in Applied Mathematics* 14 (1993): 39–50. On the real-valued functions, see "Recursion Theory on the Reals and Continuous-Time Computation," *Theoretical Computer Science* 162 (1996): 23–44.

a function by a constant, adding a constant to a function, adding functions and integrating functions.

In engineering practice, the GPAC model is performed with the use of analog electronic circuits (in short: AEC) that process data on the basis of adequately configured operational amplifiers. I will present how they work in example 2.

Example 2. In technical terms, every AEC is composed of a finite number of *basic* systems which create nodes of a net and electrical connections which connect nodes and conduct analog signals. Every basic system is an adequately configured (by adding external components) *operational amplifier*, which physically performs one simple mathematical operation, for example, summing, multiplying, comparing, differentiating or integrating.[20]

Designed for a specific purpose (which can be, for example, finding solutions of a differential equation), an AEC functions in the following way: 1) the AEC input (which is one component or more) receives a particular *function* (for example, sinusoidal) in a continuous, real time fashion; 2) the continuously performed values of the function are carried to the subsequent AEC components (sometimes in a parallel way) and modified there; 3) the result (for example, the results of integrating the following "fragments" of the input function) is also successively generated in real time at the output.

At any time, the user of AEC can *measure* the output signal and obtain a single result that he is interested in (for example, an integral result), observe the functions that are generated (for example, on the screen) and interpret them systematically as some functional results (which can be, for example, the results of differential equations).

Designing the AECs to solve specific problems consists in combining adequately any number of freely chosen basic circuits (such as an adder or a comparator). Thus, the "programme" which performs a defined function is the physical structure of a circuit.

Although the above-mentioned example (of an electronic realisation of a model) refers to specific technical solutions that were and still are used in practice, the existence of these solutions should not be treated as an argument for the actual continuity of computations

[20] Included in every basic system, an operational amplifier is a system with two inputs and one output. An ideal operational amplifier (a theoretical circuit) amplifies the signal to the extent that equals the infinity (in a real circuit, it is about maximally high amplification being almost approximate to the infinity). Without any external components, an amplifier works in the following way: the output signal is produced by multiplying the first input signal by "plus infinity" and adding it to the second input signal multiplied by "minus infinity." If we insert resistors, the "infinity" will be replaced by a concrete quantity, which can be represented by a real number. Configurations that include capacitors (capacitance) realise the process of integration and differentiation. Cf. Z. Kulka Z., M. Nadachowski, *Wzmacniacze operacyjne i ich zastosowanie* [Operational amplifiers and their applications], part 2, Warsaw 1982. I would like to thank Jarosław Sokołowski, a participant of philosophical and informational seminars organised at the Warsaw University of Technology, for valuable information on analog electronic circuits.

described by the GPAC model and other related models, since two doubts appear. Firstly, in every electronic circuit, the measurement of resulting values is always taken with some finite accuracy, which can unquestionably discredit the result (described theoretically as a continuous object). Secondly, it is unknown whether basic “continuous” operations, performed by, for example, operational amplifiers, are not actually, at a sufficiently low level of description, discrete operations.

2. Particular methodological and philosophical issues

There exist several interesting philosophical questions issuing from the formal investigations presented above.

2.1. The empirization of computations and their reliability

The first issue is directly related to the first way of understanding analogicity (AN-A) and concerns the reliability (in a narrow sense: accuracy) of computations based on the principle of natural analogy. As already indicated in point 1.2., the mathematical reliability of procedures of this type (that is efficiently using them to perform some mathematical operations) must depend on the level of *adequacy of a theory* that connects formulas and results of computations with physical reality (more accurately: processes that perform these computations).

The mentioned theory – being the result of *idealisation* procedure, typical of empirical sciences, which consists in examining phenomena ignoring factors that are recognised as unimportant – is never hundred percent appropriate.[21] Thus, if the results of mathematical operations are sought directly in the reality that is described by the theory (for example, through experiment, measurement, etc.), they must be distorted by the very same factors that have been omitted during idealisation. Metaphorically speaking: the procedure of idealisation works both ways. It allows to create a cognitively effective theory but trying to realise theoretical computations by referring to (not idealised) reality, it must cause *mistakes*.

Performing computations with the use of empirical method requires an additional reflection on types of theories that *justify* this method well enough. In the light of remarks made in point 1.2, the problem of choosing between physical and biological theories arises. That is to say: should natural computations, which are more and more often used, be justified

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[21] See W. Krajewski, *Prawa nauki*.

only within *physics* (as it happens in the case of quantum computations and traditional analog techniques), or is referring to *biology* equally valid?

On the one hand, there is no doubt that animate systems, examined by biologists in their natural environment, demonstrate a huge level of effectiveness in solving problems (mainly adaptation problems). From the point of view of computer science, they can be treated as a "ready-to-use," sophisticated product of natural evolution. On the other hand, biological theories are far less formalized than physical theories, hence they do not provide that good justification of computational effectiveness of (in this case: living) systems used. Perhaps, in the case of computer science that is oriented towards biology, we are doomed to create solutions by the process trial and error that is very unreliable (and can be partly justified by the fact that mechanisms and systems described by biology simply work well in nature).

2.2. The physical realisation of continuous (hyper)computations

Another methodological issue is related to analog computations in the sense of AN-C, that is continuous. Theoretical analyses indicate that computations of this type – described, for example, with the use of a model of recursive real-valued functions – have the status of *hypercomputations*.[22] This means that they allow solving problems that are out of reach for digital techniques which are formally expressed by the model of universal Turing machine.[23] One of such problems is the issue of solvability of diophantine equations.[24]

Although the theory of continuous computations does predict that they have *higher computational power* than digital techniques, the important question about practical *implementability* of continuous computations arises.[25] That is to say: if the physical world, the source of real data carriers and processes to process data, was discrete (quantised), we would never be able to perform any analog-continuous computations.

The question about the separateness of the mind (or even the mind/brain understood as a biological system) from the physical world, to which real digital automaton belong, is related to this issue. Perhaps the fact that the mind's computational power is higher than the power of

[22] In the definitional (semantic) sense, continuous computations are not equal to digital computations. This is because their constitutive feature, that is *continuity*, constitute a vital extension of one of constitutive features of traditional Turing computations, that is discreteness (see also footnote 5).
[23] Cf. J. Mycka, *Obliczenia dyskretne i ciągłe*, pp. 247–260.
[24] Cf. D. Harel, *Rzecz o istocie informatyki. Algorytmika* [On the essence of computer science. Algorithmics], Warsaw 2000.
[25] Cf. J. Mycka, *Obliczenia dyskretne i ciągłe*, pp. 247–260.

digital machines – which, according to some people, is proven by the observed ability of the mind to solve intuitively difficult mathematical problems[26] – can be justified with the *continuity* of mental sphere (or even the continuity of nervous system).[27]

2.3. Universal analog machines

Probably the most crucial difference between analog computations (of both types) and digital techniques consists in the fact that only in the case of the latter, there is a *universal executive programme* that allows executing programmes provided from the outside on a digital machine correctly. On a mathematical level, a universal Turing machine corresponds to it.[28]

In the case of analog-continuous computations, there are certain models of computations defined (such as the GPAC or EAC models), however, they "imply" that various specialised analog systems have to be constructed for different problems. Even if a model defines a minimal set of computations/elementary operations (whose different combinations are enough to realise any complex computations), there is still a question about a "universal" automaton that is able to simulate any specialised circuit (treated as a programme provided from the outside).

If such an automaton existed, it would be an analog equivalent of the *universal Turing machine* (UTM). The input would receive a signal that encodes an analog circuit U (its structure) and an input signal S of the circuit C. On this basis, the universal automata would simulate perfectly the operation of the circuit C for the signal S. In other words: for any possible circuit C and every possible input signals/data S_D, the *universal circuit* UC would generate the same resulting signal S_R as the circuit C would generate for the input S_D.

In the theory of analog-continuous computations, the described circuit is not defined, thus the question whether it is at all possible arises.[29] If this is not, then what are the theoretical arguments behind it?

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[26] Cf. W. Marciszewski, *Racjonalistyczny optymizm poznawczy w Gödlowskiej wizji dynamiki wiedzy* [Rationalist cognitive optimism in Gödel's vision of dynamics of knowledge], in *Przewodnik po epistemologii* [A companion to epistemology], ed. R. Ziemińska, Kraków 2013.

[27] The importance of the argument about "the continuity of nervous system" was also noticed by Alan Turing himself while he was writing about the hypothetical biological superiority of established mind over digital machines. Cf. A. M. Turing, "Computing Machinery and Intelligence," *Mind* 49 (1950): 433–460.

[28] See A. M. Turing, "On Computable Numbers, with an Application to the Entscheidungsproblem," *Proc. Lond. Math. Soc.* 42 (1936): 230–265.

[29] In case of a negative answer, analog-continuous computations would be characterised by irremovable weakness – especially in comparison to digital computations. Although analog circuits provided theoretically higher computational power than digital circuits, a programmable computer would not exist in their case. For every problem or a group of problems, a separate analog circuit would have to be designed.

In relation to analog computations of the second type, called here in short *analogous*, the problem of the universal machine cannot be presented in the same way as it has already been done above, because such computations are, by definition, of *local* character. Let me remind that in their case, natural analoga to particular types of mathematical operations are searched for. However, one could ask about the *maximalist* physical (more broadly: natural) theory, on the basis of which natural analoga of all mathematical operations (or at least those that are useful) could be found. The existence of such a theory – especially in the light of results of research conducted by K. Gödel or, in our times, G. Chaitin[30] – is indeed truly doubtful.

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[30] See G. Chaitin, *The Limits of Mathematics*, London 2003.

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